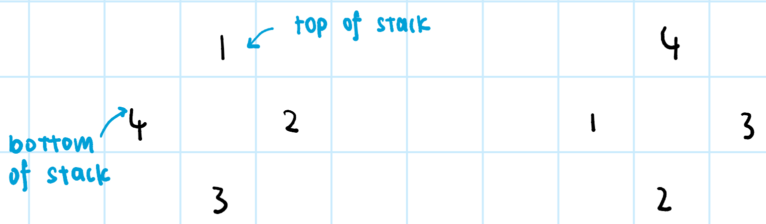
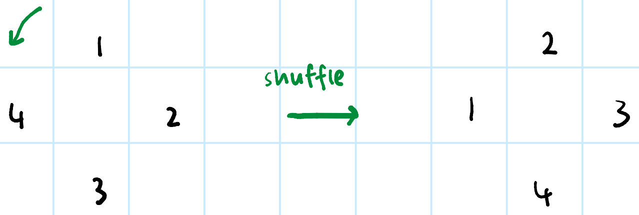
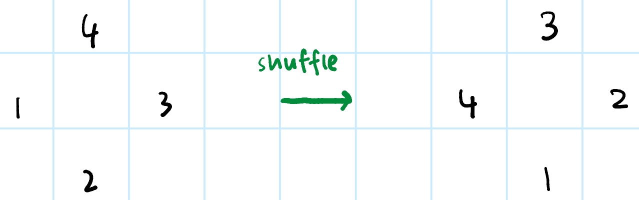
Card Trick



What does a shuffle do?





Suppose we shuffle the left deck l times, right deck r times.

$$l + r = 11.$$

Can we find an expression for the top card of each deck in terms of l & r?

↳ $l = 2$: $\qquad$ $r = 3$:

$l = 6$: $\qquad$ $r = 11$:

$T_L \equiv l + 1 \pmod 4$ $\qquad$ $T_R \equiv 4 - r \equiv -r \pmod 4$

$$\begin{aligned} T_L &\equiv l + 1 \\ &\equiv (11 - r) + 1 \\ &\equiv 12 - r \\ &\equiv 0 - r \\ &\equiv T_R \pmod 4 \end{aligned}$$

After we remove the top card...

a_1 $\qquad\qquad$ a_3

a_3 $\quad$ a_2 $\qquad\qquad$ a_1 $\quad$ a_2

Similarly, we shuffle left pile l_1 times, right pile r_1 times.

$l_1 + r_1 = 11$

↳ $l_1 = 1$: $\qquad\qquad$ $r_1 = 2$:

$l_1 = 5$: $\qquad\qquad$ $r_1 = 6$:

$T_L = a_k$, $k \equiv l_1 + 1 \pmod 3$ $\qquad\qquad$ $T_R = a_i$, $i \equiv 3 - r_1 \equiv -r_1 \pmod 3$

$k \equiv l + 1$

$\equiv (11 - r) + 1 \equiv 12 - r \equiv -r \equiv i \pmod 3$

Last shuffle (2 cards in each pile):

b_1 $\qquad\qquad$ b_2

b_2 $\qquad\qquad$ b_1

l_2 shuffles $\qquad\qquad$ r_2 shuffles

$T_L = b_k$, $k \equiv l_2 + 1 \pmod 2$ $\qquad\qquad$ $T_R = b_i$, $i \equiv 2 - r_2 \equiv -r_2 \pmod 2$

$k \equiv l_2 + 1$

$\equiv (11 - r_2) + 1 \equiv 12 - r_2 \equiv -r_2 \equiv i \pmod 2$

What's so special about 11?

$11 \equiv -1 \pmod 4$

$\equiv -1 \pmod 3$

$\equiv -1 \pmod 2$

In general... $\quad$ m cards in each pile

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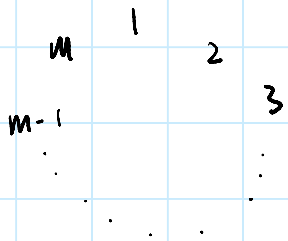

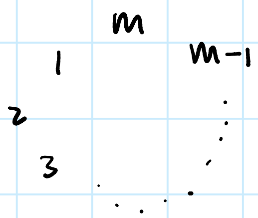

Word with N letters such that
$$
\begin{aligned}
N &\equiv -1 \pmod{m} \\
&\equiv -1 \pmod{m-1} \\
&\cdots \\
&\equiv -1 \pmod{3} \\
&\equiv -1 \pmod{2}
\end{aligned}
$$